Multi-stage lot sizing models with imperfect processes and inspection errors

M. BEN-DAYA and A. RAHIM

Keywords multi-stage systems, lot sizing, imperfect processes, inspection errors.

Abstract. This paper deals with multi-stage lotsizing models for imperfect production processes. The effect of imperfect quality on lotsizing decisions and effect of inspection errors are taken into consideration in the proposed models. Numerical examples are presented for illustration purposes. The developed models are very helpful for justifying quality assurance and quality improvement efforts.

1. Introduction

In the production and inventory analysis literature, the economic production quantity (EPQ) has been studied under various conditions (Hax and Candea 1984, Silver and Peterson 1985). The EPQ models have been embellished considerably through the incorporation of a number of real world considerations. Banerjee and Burton (1990) developed a general approach for tackling the EPQ problem in single and multistage production systems taking into account work-in-process (WIP) inventories. They also incorporated the notion of gradual transformation of input to output at each production stage. The developed models assume that the production processes never fail and they always produce items of acceptable quality. However, a more realistic situation is one in which the quality is not always acceptable because the condition of the production process may deteriorate with time. Rosenblatt and Lee (1986) studied the effect of substandard quality, due to a deteriorating process, on lot sizing decisions. In particular, they show that the optimal production cycle is shorter than that of the classical EPQ model for the single stage lot sizing problem. The issue of the effect of imperfect processes has been addressed by many authors including Porteus (1986), Lee and Rosenblatt (1987), and Groenevelt et al. (1992), among others. Rahim (1994), and Rahim and Ben-Daya (1998) looked at the effect of EPQ on the...
economic design of $\bar{x}$ control chart. They developed an integrated model for the inventory and quality control problems for a class of deteriorating processes where the in-control period follows a general probability distribution with increasing hazard rate.

The purpose of this paper is to model the effect of imperfect production processes on lot sizing decisions in the context of multi-stage production systems. This issue does not seem to have been adequately addressed in the literature for the multi-stage case. Also, in a multi-stage framework and in the presence of imperfect production processes, defective items should not be passed to subsequent stages in order to avoid unnecessary processing and reduce waste. In this case, the issue of inspection errors, if present, must also be addressed. Using the approach proposed by Banerjee and Burton (1990) for tackling the lot sizing problem in multi-stage production systems, we develop more general models which take into consideration the effect of imperfect production processes and also the effect of errors committed at the various production stages when screening non-conforming items. Numerical examples are provided to illustrate the various models developed.

This paper is organized as follows. In the next section, we define the problem and develop the necessary notations and assumptions. The Banerjee and Burton (1990) multi-stage lot sizing model is presented in Section 3. Models incorporating imperfect production processes are introduced in Section 4. In Section 5, errors are integrated in the proposed models. Examples illustrating the various models are presented in Section 6. Finally, Section 7 contains a summary of the paper and some concluding remarks.

2. Problem definition

Consider a multi-stage production system producing a single end product in $n$ stages. It is assumed that demand and production rates at all stages are constant and the production rate at any stage exceeds the demand rate, i.e. $P_j \geq D_j$, $j = 1, 2, \ldots, n$. No backlogging is permitted. Beyond the first stage, the work-in-process (WIP) at any subsequent stage is characterized by a single semi-finished item, until it becomes the final product after the last production stage.

The inventory items in the multi-stage system are arranged in a hierarchical manner. The final product comprises the topmost level (level 1), intermediate products at various prior stages occupy the subsequent levels and raw material and other inputs lie at the lowest level (level $n+1$). The first manufacturing stage, $(n+1)$th level items are processed to obtain the WIP at level $n$, and so on, until at stage $n$ the level 2 semi-finished item is transformed to the final product. Each unit of production is passed to the next stage as soon as processing at the current stage is completed, without waiting for the completion of the whole batch. These production systems are not uncommon in practice. Consequently, in the presence of perfect processes and no inspection errors, a uniform lot size of $Q$ units is processed at any production stage.

If the process is imperfect, i.e. may deteriorate with time, it is assumed that at each production stage the process starts in the in-control state producing items of perfect or acceptable quality. However, the process may shift to the out-of-control state after a random time with known probability distribution. Once in the out-of-control state, the process starts producing a fixed percentage of defective items and stays in this state until the end of the production run.

In the presence of deteriorating processes, defective items must be screened so that they are not passed to subsequent stages. Errors-free models and models considering inspection errors will be developed.

The following notation will be used.

- $n$ number of stages
- $Q$ production lot size for the end product
- $Q_j$ production lot size for the product at the $j$th level
- $D$ demand rate of the end product (units/unit time)
- $P_j$ production rate (units/unit time) at which the $(j+1)$th level item is converted to the $j$th level inventory item
- $P_j$ production cost excluding setup at the $j$th level
- $A_j$ setup cost at the $j$th level
- $r$ fractional inventory carrying cost in $\$/unit time
- $C_j$ production cost excluding setup at the $j$th level
- $S_j$ unit cost of producing a defective item at the $j$th level
- $\alpha_j$ fraction of defective units
- $N_j$ expected number of defective items produced at the $j$th level
- $I_j$ expected average inventory level at the $j$th level
- $t_j$ production time per cycle at the $j$th level
- $F_j$ process shift distribution at the $j$th level
- $f_j$ probability density function of the time to shift at the $j$th level
- $ETC$ expected total cost

The following three models are developed in the next three sections:

1. Multi-stage lot sizing model with perfect production processes;
2. Multi-stage lot sizing model with deteriorating processes;
Multi-stage lot sizing model with imperfect production processes and inspection errors.

3. Lot sizing models with perfect production processes

This section is based on Banerjee and Burton (1990), and is included here so that this paper is self-contained. Consider first the case of two production stages. The inventory levels at different levels are shown in Figure 1 for the case \( P_1 \leq P_2 \) and in Figure 2 for the case \( P_1 \geq P_2 \). Note that a uniform lot size of \( Q \) units is processed at any production stage.

(1) Case 1: \( P_1 \leq P_2 \). The average inventories at the three levels of the two-stage production systems are given by:

\[
I_1 = \frac{Q}{2} \left( 1 - \frac{D}{P_1} \right) \quad (1)
\]

\[
I_2 = D \frac{Q (P_2 - P_1)}{2 P_1 P_2} \quad (2)
\]

\[
I_3 = D \frac{Q}{2} P_2. \quad (3)
\]

(2) Case 2: \( P_1 \geq P_2 \). The average inventories at levels 1 and 3 are the same as in Case 1, but \( I_2 \) is given by:

\[
I_2 = D \frac{Q (P_1 - P_2)}{2 P_1 P_2} \quad (5)
\]

From equations (2) and (5), the following expression for \( I_2 \) holds regardless of the relative magnitude of \( P_1 \) and \( P_2 \).

\[
I_2 = D \frac{Q |P_1 - P_2|}{2 P_1 P_2} \quad (6)
\]

The expected total cost per cycle with perfect production processes is the sum of the setup costs and inventory holding costs, and is given by:

\[
ETC_p(Q) = \frac{D}{Q} (A_1 + A_2 + A_3)
\]

\[
+ \frac{Q \tau}{2} \left[ C_1 \left( 1 - \frac{D}{P_1} \right) + C_2 D \frac{|P_1 - P_2|}{P_1 P_2} + C_3 \frac{D}{P_2} \right] \quad (7)
\]

It can easily be verified that the optimal lot size is given by:

![Figure 1. Inventory time plots for a two-stage system; Case 1: \( P_1 \leq P_2 \).](image-url)
These results can be readily generalized to the $n$-stage case. The cost function and optimal lot size are as follows.

\[ Q^* = \left[ \frac{2D(A_1 + A_2 + A_3)}{r C_1 \left(1 - \frac{D}{P_1}\right) + C_2D \left|\frac{P_1 - P_2}{P_1P_2} + C_3 \frac{D}{P_2}\right|} \right]^{1/2} \]  

(8)

These results can be readily generalized to the $n$-stage case. The cost function and optimal lot size are as follows.

\[ ET C_p(Q) = \frac{D}{Q} \sum_{j=1}^{n+1} A_j + \frac{Qr}{2} \]

\[ \times \left[ C_1 \left(1 - \frac{D}{P_1}\right) + D \sum_{j=2}^{n} \left|\frac{P_j - P_{j-1}}{P_j P_{j-1}}\right| \right] \]

\[ + C_{n+1} \frac{D}{P_n} \]  

(9)

and

\[ Q^* = \left[ \frac{2D \sum_{j=1}^{n+1} A_j}{r C_1 \left(1 - \frac{D}{P_1}\right) + D \sum_{j=2}^{n} \left|\frac{P_j - P_{j-1}}{P_j P_{j-1}}\right| + C_{n+1} \frac{D}{P_n}} \right]^{1/2} \]

(10)

4. Lot sizing models with imperfect production processes

In this section, we assume that the quality of the output of the various stages is not perfect. For each cycle, at the $j$th level, the production process may shift at a random time to the out-of-control state and starts producing a fixed fraction $\alpha_j$ of defective items. It is assumed that defective items are screened out before passing to the next production stage, and that the inspection process is error free. This assumption will be relaxed in the next section where different types of errors will be incorporated into an extended model.

The expected number of defective items produced at the $j$th level is given by:

\[ N_j = \int_0^{t_j} \alpha_j f_j(i) \, di \]  

(11)

where $t$ is the elapsed time for which the process at the $j$th level remains in the in-control state before a shift occurs and $t_j$ is the length of the production run at level $j$. It is assumed that once a shift occurs, the process stays in the out-of-control state until the end of the production cycle.
With each setup, the process is brought back to the in-control state. If, at the \( j \)th level, the time to shift distribution is exponential with mean \( \theta_j \), i.e. \( f_j(t) = 1/\theta_j e^{-t/\theta_j} \), then

\[
N_j = \alpha_j P_j (t_j - \theta_j + \theta_j e^{-t_j/\theta_j}) \tag{12}
\]

Using the approximation \( e^{-t_j/\theta_j} \approx 1 - t_j/\theta_j + (-t_j/\theta_j)^2/2 \), we obtain the following expression for \( N_j \):

\[
N_j = \frac{\alpha_j P_j t_j}{2\theta_j} \tag{13}
\]

\[
= \frac{\alpha_j Q_j^2}{2P_j \theta_j} \tag{14}
\]

The last equality follows from the fact that \( t_j = Q_j/P_j \).

Note that a lot of \( Q_j \) good products are required for the final product. Defective items at each level are removed and not passed to the following stage. Hence, the lot size \( Q_j \) at the intermediate level \( j \) can be obtained recursively as follows:

\[
Q_{j-1} = Q_j - N_j
\tag{15}
\]

or

\[
Q_j = Q_{j-1} + \frac{\alpha_j Q_{j-1}^2}{2P_j \theta_j} \tag{16}
\]

Solving for \( Q_j \), we obtain

\[
Q_j = \frac{P_j - \sqrt{P_j^2 - (2\alpha_j P_j Q_{j-1})/\theta_j}}{\alpha_j / \theta_j} \tag{17}
\]

The other root of equation (4) is meaningless since \( Q_j \to \infty \) as \( \alpha_j \to 0 \) in this case. However, it is easy to verify that \( Q_j \to Q_{j-1} \) as \( \alpha_j \to 0 \) in equation (17), as expected.

Next, we develop the complete cost model in the presence of imperfect production processes. The expected total cost includes setup costs, inventory holding costs and quality related cost due to the production of defective items if a process shifts to the out of control state.

1. **Setup costs:**
   The setup cost per unit time for an \( n \)-stage system is:
   \[
   SC = \frac{D}{Q} \sum_{j=1}^{n} A_j
   \tag{18}
   \]

2. **Inventory holding cost:**
   Consider first the case of two production stages. The inventory levels at different levels, for \( p_1 \leq p_2 \) and \( p_1 \geq p_2 \), are similar to those shown in figures 1 and 2, in the sense that they have the same shape. The difference is that the lot sizes in levels 1 and 2 are different and defective items produced at level 2 are screened out and not kept in the inventory between levels 2 and 1.

   a) **Case 1:** \( p_1 \leq p_2 \). The average inventories at the three levels of the two-stage production systems are as follows:
      i) **Level 1:** At this level, the maximum inventory level is
      \[
      (p_1 - D)q - N_1 = (p_1 - D)q - Q_1 + Q_2
      \]
      That is the inventory accumulated during a period of length \( q \) at the rate \( p_1 - D \) minus the defective items which have been screened out. Consequently, \( I_1 \) is given by (see figure 1).
      \[
      I_1 = \frac{1}{2} \left[ Q - \frac{D}{p_1} Q_1 \right] \tag{19}
      \]
      ii) **Level 2:** From figure 2, and using an argument similar to that used to derive \( I_1 \), \( I_2 \) is given by:
      \[
      I_2 = \frac{D Q_2}{2Q} \left[ \frac{Q_1 - Q_2}{p_1 - p_2} \right] \tag{20}
      \]
      iii) **Level 3:** Level 3 items are assumed to be of perfect quality. Hence, \( I_3 \) is given by:
      \[
      I_3 = \frac{D Q_2}{2Q P_2} \tag{21}
      \]

b) **Case 2:** \( p_1 \geq p_2 \). The average inventories at levels 1 and 3 are the same as in Case 1, but \( I_2 \) can be derived as follows.

   From figure 2, note the maximum inventory level is equal to the accumulated inventory during the time \( (t_2 - q) \) at a rate of \( p_2 \) minus the expected number of defective items produced during the time \( t_2 - q \) which is approximated by \( (\alpha_2 p_2 (t_2 - q)^2)/(2\theta_2) \) using equation (13). After simplification, we obtain the following expression for \( I_2 \):

   \[
   I_2 = \frac{D Q_2}{2Q} \left[ \frac{Q_2 - Q_1}{p_2} \right] \left[ 1 - \frac{\alpha_2 p_2 (Q_2 - Q_1)}{2\theta_2 (p_2 - p_1)} \right] \tag{22}
   \]

   The above results can be generalized to the \( n \)-stage case as follows.
The expected total cost per unit time is simply the sum of equations (18), (14) and (17), that is

\[
ETC_i(Q) = \frac{D}{Q} \sum_{j=1}^{i+1} A_j + r \sum_{j=1}^{i+1} C_j I_j + \frac{D}{Q} \sum_{j=1}^{n} \alpha_j Q_j^2 \frac{Q_j}{2P_j \theta_j} \tag{28}
\]

5. Integration of inspection errors

In this section, we take into consideration inspection errors. While screening products at any stage to remove defective items, if any, two types of errors may be committed:

1. Rejecting a conforming item at the \( j \)-th level, with probability \( E_{1,j} \). This is known as Type I error.
2. Accepting a non-conforming item at the \( j \)-th level, with probability \( E_{2,j} \). This is known as Type II error.

The following additional notation is needed:

\( s_{1,j} \) cost of incorrectly rejecting a conforming item at level \( j \);
\( s_{2,j} \) cost of incorrectly accepting a non-conforming item at level \( j \);
\( s_{3,j} \) cost of correctly rejecting a non-conforming item at level \( j \).

Clearly, the expected number of defective items produced at level \( j \) is \( N_j \) and the expected number of good items is \( Q_j - N_j \). Hence, from figure 3, it can be seen that

\[
Q_j - 1 = (Q_j - N_j)(1 - E_{1,j}) + N_j E_{2,j} \tag{29}
\]

\[
= Q_j (1 - E_{1,j}) - (1 - E_{1,j} - E_{2,j}) \alpha_j Q_j^2 \frac{Q_j}{2P_j \theta_j} \tag{30}
\]
The last equality follows from equation (14). Solving this quadratic equation for \( Q_j \), we obtain:

\[
Q_j = \frac{\frac{2\alpha(P_jQ_j - (1 - E_{i_{j-1}} - E_{i_j}))}{\theta_j}}{\alpha(1 - E_{1_{i_j}} - E_{2_{i_j}})}
\]

(31)

Again, note that the other root is meaningless since \( Q_j \to 0 \) as \( \alpha \to 0 \) in this case. However, it is easy to verify that \( Q_j \to Q_{\infty} \) as \( \alpha \to 0 \) in equation (31), as expected.

Next, we develop the complete cost model for determining the economic lot sizes for a multi-stage production system in the presence of imperfect production processes and incorporating inspection errors.

The expected total cost includes setup costs, inventory holding costs and quality related cost due to the production of defective items if a process shifts to the out-of-control state and the costs due to inspection errors.

The setup cost is not changed and is given by equation (18). However, the inventory holding cost and quality related costs are derived as follows.

1. **Inventory holding cost:**

   Here, we also first consider the case of two production stages and deal with the two cases \( P_1 \geq P_2 \) and \( P_1 \leq P_2 \), separately.

   In this case, the \( Q_j \)'s at different levels are related by equation (31), and the expected number of defective items that are actually rejected at level \( j \) is given by:

   \[
   N_j' = N_j(1 - E_{2_{j-1}}) + (Q_j - N_j)E_{1_{j}}
   \]

   (32)

   which is the sum of correctly rejected defective items and incorrectly rejected good items.

   (a) **Case 1:** \( P_1 \leq P_2 \). The average inventories at the three levels of the two-stage production systems are as follows:

   (i) **Level 1:** At this level, the maximum inventory level \( I_{1_{max}} \) is

   \[
   I_{1_{max}} = (P_1 - D)Q_1 - N_1'
   \]

   (33)

   The last equality follows from that fact the \( N_1' \) is given by equation (32) and that \( Q = (Q_1 - N_1')(1 - E_{1_{i_1}}) + N_1E_{2_{i_1}} \) (see figure 3). Since the length of the inventory cycle at level 1 is \( Q/P_1 \), the average inventory at level 1 is given by:

   \[
   I_1 = \frac{1}{2} \left[ Q - \frac{D}{P_1}Q_1 \right]
   \]

   (34)

   (ii) **Levels 2 and 3:** Using similar arguments, it can be shown that

   \[
   I_2 = \frac{D}{2} \left[ \frac{Q_1 - Q_2}{P_1 - P_2} \right]
   \]

   (35)

   \[
   I_3 = \frac{D}{2} \left( \frac{Q_2}{P_2} \right)
   \]

   (36)

   (b) **Case 2:** \( P_1 \geq P_2 \). The average inventories at levels 1 and 3 are the same as in Case 1, but \( I_2 \) can be derived as follows.

   From figure 2, note that the maximum inventory level, \( I_{2_{max}} \), is equal to the accumulated inventory during the time \( t_2 = t_2 - t_1 \) at a rate of \( P_2 \) minus the expected number of defective items actually rejected during the time \( t_2 - t_1 \) taking into account inspection errors, call it \( N_{2_{c}}' \). Hence,

   \[
   I_{2_{max}} = P_2(t_2 - t_1) - N_{2_{c}}'
   \]

   (37)

   Using equation (13), \( N_{2_{c}}' \) is given by:

   \[
   N_{2_{c}}' = \frac{\alpha P_2 (t_2 - t_1)^2}{2 \theta_2} (1 - E_{1_{i_2}} - E_{2_{i_2}})
   \]

   \[
   + P_2(t_2 - t_1)E_{1_{i_2}}
   \]

   (38)

   Substituting equation (38) into equation (37) and noting that

   \[
   I_2 = \frac{1}{2} \frac{I_{2_{max}}}{Q/P_1}
   \]

   we obtain after simplification the following expression for \( I_2 \).

   \[
   I_2 = \frac{D}{2} \left( \frac{Q_2 - Q_1}{P_2 - P_1} \right)
   \]

   \[
   \times \left[ 1 - E_{1_{i_2}} - (1 - E_{1_{i_2}} - E_{2_{i_2}}) \frac{\alpha P_2}{2 \theta_2} \left( \frac{Q_2 - Q_1}{P_2 - P_1} \right) \right]
   \]

   (39)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>3</td>
<td>( c_3 )</td>
<td>2</td>
</tr>
<tr>
<td>( D )</td>
<td>10000</td>
<td>( c_4 )</td>
<td>1</td>
</tr>
<tr>
<td>( P_1 )</td>
<td>50000</td>
<td>( \alpha )</td>
<td>0.25</td>
</tr>
<tr>
<td>( P_2 )</td>
<td>40000</td>
<td>( \theta_1 )</td>
<td>0.10</td>
</tr>
<tr>
<td>( P_3 )</td>
<td>100000</td>
<td>( \theta_j )</td>
<td>0.05</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>100</td>
<td>( s_{1,j} )</td>
<td>0.5 ( c_j )</td>
</tr>
<tr>
<td>( A_2 )</td>
<td>35</td>
<td>( s_{2,j} )</td>
<td>( c_j )</td>
</tr>
<tr>
<td>( A_3 )</td>
<td>20</td>
<td>( s_{2,1} )</td>
<td>1.5 ( c_1 )</td>
</tr>
<tr>
<td>( A_4 )</td>
<td>15</td>
<td>( s_{2,2} )</td>
<td>( c_1 )</td>
</tr>
<tr>
<td>( C_1 )</td>
<td>10</td>
<td>( s_{2,3} )</td>
<td>( c_2 )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Notice that equation (39) reduces to equation (22) when the inspection process is perfect, i.e. 
\( E_{1,2} = E_{2,2} = 0 \).

The above results can be generalized to the \( n \)-stage case as follows.

\[
I_1 = \frac{1}{2} \left( Q - \frac{D}{P_1} Q \right)
\]

\[
I_j = \begin{cases} 
\frac{D}{2Q} \left( \frac{Q}{P_j} - \frac{Q}{P_{j-1}} \right) \\
\left( 1 - E_{1,j} - (1 - E_{1,j} - E_{2,j}) \frac{\alpha_j}{2P_j} \right) \\
\times \left( P_j - P_{j-1} \right) \\
\frac{D}{2Q} \left( \frac{Q}{P_{j-1}} - \frac{Q}{P_j} \right)
\end{cases}
\]

\[
I_{s+1} = \frac{D \alpha_j^2}{2QP_n} \tag{42}
\]

Consequently, the expected inventory holding cost per unit time is given by:

\[
HC = r \sum_{j=1}^{s+1} C_jI_j \tag{43}
\]

where \( I_1, I_2, \ldots, I_{s+1} \) are given by equations (40)–(42).

(2) Quality related cost:

Using equation (14) and figure 3, the expected quality related cost per cycle is given by:

\[
QC = \frac{D}{Q} \sum_{j=1}^{n} \left[ s_{1,j}E_{1,j} \left( Q_j - \frac{\alpha_j Q_j^2}{2P_j \theta_j} \right) + s_{2,j}E_{2,j} \frac{\alpha_j Q_j^2}{2P_j \theta_j} \left( 1 - E_{2,j} \right) \right] \tag{44}
\]

It is assumed that raw materials and input products are of perfect quality.

The expected total cost per unit time is simply the sum of equations (18), (43) and (44).

6. Numerical examples

Numerical examples are presented to illustrate the models developed in Sections 4 and 5. The optimum values of the lot sizes at different stages, the total expected cost are obtained. A FORTRAN code implementing the Hooke and Jeeves optimization procedure is used to find these solutions. Consider the data shown in Table 1.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( Q )</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( Q_3 )</th>
<th>( ETC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>759</td>
<td>764</td>
<td>771</td>
<td>773</td>
<td>4092.31</td>
</tr>
<tr>
<td>0.10</td>
<td>584</td>
<td>590</td>
<td>598</td>
<td>601</td>
<td>5282.49</td>
</tr>
<tr>
<td>0.25</td>
<td>391</td>
<td>398</td>
<td>408</td>
<td>412</td>
<td>7901.49</td>
</tr>
<tr>
<td>0.40</td>
<td>308</td>
<td>315</td>
<td>325</td>
<td>329</td>
<td>9891.39</td>
</tr>
</tbody>
</table>

Table 2. Results for various proposed models.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( Q )</th>
<th>( Q_1 )</th>
<th>( Q_2 )</th>
<th>( Q_3 )</th>
<th>( ETC )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>576</td>
<td>588</td>
<td>602</td>
<td>608</td>
<td>6163.72</td>
</tr>
<tr>
<td>0.10</td>
<td>577</td>
<td>589</td>
<td>603</td>
<td>612</td>
<td>6224.74</td>
</tr>
<tr>
<td>0.50</td>
<td>562</td>
<td>573</td>
<td>586</td>
<td>620</td>
<td>6728.54</td>
</tr>
</tbody>
</table>

7. Conclusion

The effect of deteriorating processes on the EPQ in a single stage received a lot of attention from researchers. However, these problems have not been adequately addressed in the context of multi-stage production systems.

In this paper, we developed multi-stage lot sizing models for imperfect production processes. The effect of inspection errors that may be committed while screening
defective items that may be produced at various stages has also been incorporated. Numerical examples have been presented to illustrate the various models developed in this paper.

It is believed that these models provide a step forward in the more realistic modelling and understanding of multi-stage production systems. Other extensions may include investigating the effect of maintenance policies on these systems. Some work in this direction is being conducted by the authors.

Acknowledgment

The authors are grateful to S.O. Duffuaa for useful discussions and would like to acknowledge the support of King Fahd University of Petroleum and Minerals.

References


Rahim, M. A., 1994, Joint determination of production quantity inspection schedule, and control chart design. IIE Transactions, 26, 2–11.
